Preparation for A Level Maths @ Wadebridge School

Bridging the gap between GCSE and A Level

Name:_______________________________________

CONTENTS

The following book is covering part of the topics needed

AS-Level Maths Head Start
Published by CGP Workbooks
ISBN: 978 1 84146 993 5
Cost: £4.95

Chapter 1: REMOVING BRACKETS

To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including

* the smiley face method
* FOIL (Fronts Outers Inners Lasts)
* using a grid.

Example:

$$\begin{aligned}(x - 2)(2x + 3) &= x(2x + 3) - 2(2x + 3)\\ &= 2x^2 + 3x - 4x - 6\\ &= 2x^2 - x - 6\end{aligned}$$

or

$$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$$

or

	x	-2
$2x$	$2x^2$	$-4x$
3	$3x$	-6

$$\begin{aligned}(2x + 3)(x - 2) &= 2x^2 + 3x - 4x - 6\\ &= 2x^2 - x - 6\end{aligned}$$

Two Special Cases

Perfect Square:

$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$

$(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$

Difference of two squares:

$(x - a)(x + a) = x^2 - a^2$

$(x - 3)(x + 3) = x^2 - 3^2$

$= x^2 - 9$

EXERCISE 1A Multiply out the following brackets and simplify.

1. $(x + 2)(x + 3)$

2. $(t - 5)(t - 2)$

3. $(2x + 3y)(3x - 4y)$

4. $4(x - 2)(x + 3)$

5. $(2y - 1)(2y + 1)$

6. $(3 + 5x)(4 - x)$

7. $(7x - 2)^2$

8. $(5y - 3)(5y + 3)$

Chapter 2: LINEAR EQUATIONS

Example 1: Solve the equation $\frac{x+1}{4}+\frac{x+2}{5}=2$

Solution:

Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator $\frac{20(x+1)}{4}+\frac{20(x+2)}{5}=40$

Step 3: Simplify the left hand side: $\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}}+\frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}}=40$

$5(x + 1) + 4(x + 2) = 40$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13 $9x = 27$

Step 7: Divide by 9: $x = 3$

Example 2: Solve the equation $x+\frac{x-2}{4}=2-\frac{3-5x}{6}$

Solution: The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x+\frac{12(x-2)}{4}=24-\frac{12(3-5x)}{6}$$

Simplify	$12x+3(x-2)=24-2(3-5x)$
Expand brackets	$12x+3x-6=24-6+10x$
Simplify	$15x-6=18+10x$
Subtract $10x$	$5x-6=18$
Add 6	$5x = 24$
Divide by 5	$x = 4.8$

Exercise 2A: Solve these equations

1) $\frac{1}{2}(x+3)=5$

2) $\frac{y}{4}+3=5-\frac{y}{3}$

3) $\frac{7x-1}{2}=13-x$

4) $2x+\frac{x-1}{2}=\frac{5x+3}{3}$

FORMING EQUATIONS

Example 8: Find three consecutive numbers so that their sum is 96.

Solution: Let the first number be n, then the second is $n + 1$ and the third is $n + 2$.

Therefore
$$n + (n + 1) + (n + 2) = 96$$
$$3n + 3 = 96$$
$$3n = 93$$
$$n = 31$$

So the numbers are 31, 32 and 33.

Exercise 2B:

1) Find 3 consecutive even numbers so that their sum is 108.

2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has.
Form an equation, letting n be the number of photographs one girl had at the **beginning**.
Hence find how many each has **now**.

Chapter 3: SIMULTANEOUS EQUATIONS

An example of a pair of simultaneous equations is $3x + 2y = 8$ ①
$5x + y = 11$ ②

In these equations, x and y stand for two numbers. We can solve these equations in order to find the values of x and y by eliminating one of the letters from the equations.

In these equations it is simplest to eliminate y. We do this by making the coefficients of y the same in both equations. This can be achieved by multiplying equation ② by 2, so that both equations contain $2y$:

$3x + 2y = 8$ ①
$10x + 2y = 22$ 2×② = ③

To eliminate the y terms, we subtract equation ③ from equation ①. We get: $7x = 14$
i.e. $x = 2$

To find y, we substitute $x = 2$ into one of the original equations. For example if we put it into ②:

$10 + y = 11$
$y = 1$

Therefore the solution is $x = 2$, $y = 1$.

Remember: You can <u>check</u> your solutions by substituting both x and y into the original equations.

Example: Solve $2x + 5y = 16$ ①
$3x - 4y = 1$ ②

Solution: We begin by getting the same number of x or y appearing in both equation. We can get $20y$ in both equations if we multiply the top equation by 4 and the bottom equation by 5:

$8x + 20y = 64$ ③
$15x - 20y = 5$ ④

As the SIGNS in front of $20y$ are DIFFERENT, we can eliminate the y terms from the equations by ADDING:

$23x = 69$ ③+④
i.e. $x = 3$

Substituting this into equation ① gives:

$6 + 5y = 16$
$5y = 10$
So… $y = 2$
The solution is $x = 3$, $y = 2$.

Exercise 3A:
Solve the pairs of simultaneous equations in the following questions:

1) $x + 2y = 7$
   $3x + 2y = 9$

2) $9x – 2y = 25$
   $4x – 5y = 7$

3) $4a + 3b = 22$
   $5a – 4b = 43$

Chapter 4: FACTORISING

Common factors

We can factorise some expressions by taking out a common factor.

Example 1: Factorise $9x^3y^2 – 18x^2y$

Solution: 9 is a common factor to both 9 and 18.
The highest power of x that is present in both expressions is x^2.
There is also a y present in both parts.
So we factorise by taking $9x^2y$ outside a bracket:
$$9x^3y^2 – 18x^2y = 9x^2y(xy – 2)$$

Example 2: Factorise $3x(2x – 1) – 4(2x – 1)$

Solution: There is a common bracket as a factor.
So we factorise by taking $(2x – 1)$ out as a factor.
The expression factorises to $(2x – 1)(3x – 4)$

Exercise 4A
Factorise each of the following

1) $3pq - 9q^2$

2) $8a^5b^2 – 12a^3b^4$

3) $5y(y – 1) + 3(y – 1)$

Factorising quadratics

Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$
The method is:
Step 1: Form two brackets $(x \; \ldots \;)(x \; \ldots \;)$
Step 2: Find two numbers that multiply to give c and add to make b. These two numbers get written at the other end of the brackets.

Example 1: Factorise $x^2 - 9x - 10$.

Solution: We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.
Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$
The method is:
<u>Step 1</u>: Find two numbers that multiply together to make *ac* and add to make *b*.
<u>Step 2</u>: Split up the *bx* term using the numbers found in step 1.
<u>Step 3</u>: Factorise the front and back pair of expressions as fully as possible.
<u>Step 4</u>: There should be a common bracket. Take this out as a common factor.

Example 2: Factorise $6x^2 + x - 12$.

Solution: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = \underbrace{6x^2 - 8x} + \underbrace{9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x - a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$
$16x^2 - 25 = (2x)^2 - 5^2 = (2x + 5)(2x - 5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x + 4)(x - 4)$
and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x + 4y)(x - 4y)$

Factorising by pairing
We can factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) - 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)

$= (2x + y)(x - 1)$

Exercise 4B
Factorise
1) $x^2 + 6x - 16$
2) $3x^2 + 5x - 2$
3) $2y^2 + 17y + 21$
4) $10x^2 + 5x - 30$
5) $4x^2 - 25$
10) $x^2 - 3x - xy + 3y^2$

Chapter 5: CHANGING THE SUBJECT OF A FORMULA

Example: The formula $C = \frac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.
We can rearrange to make F the subject.

$$C = \frac{5(F-32)}{9}$$

Multiply by 9 $\quad 9C = 5(F-32)$ (this removes the fraction)

Expand the brackets $\quad 9C = 5F - 160$

Add 160 to both sides $\quad 9C + 160 = 5F$

Divide both sides by 5 $\quad \frac{9C+160}{5} = F$

Therefore the required rearrangement is $F = \frac{9C+160}{5}$.

Example 2: Make a the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Solution: $\quad t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Multiply by 4 $\quad 4t = \sqrt{\frac{5a}{h}}$

Square both sides $\quad 16t^2 = \frac{5a}{h}$

Multiply by h: $\quad 16t^2 h = 5a$

Divide by 5: $\quad \frac{16t^2 h}{5} = a$

Exercise 5A

Make x the subject of each of these formulae:

1) $y = 7x - 1$

2) $y = \frac{x+5}{4}$

3) $4y = \frac{x}{3} - 2$

4) $y = \frac{4(3x-5)}{9}$

5) $P = \sqrt{\frac{2t}{g}}$

More difficult examples

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In these questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

Example 3: Make W the subject of the formula $T - W = \dfrac{Wa}{2b}$

Solution: This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by $2b$: $2bT - 2bW = Wa$

Add $2bW$ to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by $a + 2b$: $W = \dfrac{2bT}{a + 2b}$

Exercise 5B

Make x the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \dfrac{2x + 3}{5x - 2}$

4) $\dfrac{x}{a} = 1 + \dfrac{x}{b}$

Chapter 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:
* factorising
* the quadratic formula

Note that not all quadratic equations can be solved by factorising. The quadratic formula can always be used however.

Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example 1 : Solve $x^2 - 3x + 2 = 0$

Factorise $(x-1)(x-2) = 0$
Either $(x-1) = 0$ or $(x-2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Example 2: Solve $x^2 - 2x = 0$

Factorise: $x(x-2) = 0$
Either $x = 0$ or $(x-2) = 0$
So $x = 0$ or $x = 2$

Method 2: Using the formula

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3: Solve the equation $2x^2 - 5 = 7 - 3x$

Solution: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$
We can then tell that $a = 2$, $b = 3$ and $c = -12$.
Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)
If we have a calculator, we can evaluate these roots to get: $x = 1.81$ or $x = -3.31$

EXERCISE 6A

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$

b) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$

c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

$$6x^2 - 5x - 4 = 0$$

4) Use the formula to solve the following equations to 3 significant figures. Some of the equations may not have solutions.

a) $x^2 + 7x + 9 = 0$

c) $4x^2 - x = 7$

e) $3x^2 + 4x + 4 = 0$

Chapter 7: INDICES

Basic rules of indices

y^4 means $y \times y \times y \times y$.

4 is called the **index** (plural: indices), **power** or **exponent** of *y*.

There are 3 basic rules of indices:

1) $a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$
2) $a^m \div a^n = a^{m-n}$ e.g. $3^8 \times 3^6 = 3^2$
3) $(a^m)^n = a^{mn}$ e.g. $\left(3^2\right)^5 = 3^{10}$

Further examples

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the numbers and multiply the *a*'s)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the numbers and multiply the *c*'s)

$24d^7 \div 3d^2 = \dfrac{24d^7}{3d^2} = 8d^5$ (divide the numbers and divide the *d* terms i.e. by subtracting the powers)

Exercise 7A
Simplify the following:

1) $b^2c \times bc^3 =$

2) $2n^6 \times (-6n^2) =$

3) $8n^8 \div 2n^3 =$

4) $d^{11} \div d^9 =$

5) $\left(a^3\right)^2 =$

6) $\left(-d^4\right)^3 =$

More complex powers

Zero index:
Recall from GCSE that

$$a^0 = 1.$$

This result is true for any non-zero number *a*.

Therefore $5^0 = 1$ $\left(\frac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

Negative powers

A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \frac{1}{a}$

Therefore $5^{-1} = \frac{1}{5}$

$$0.25^{-1} = \frac{1}{0.25} = 4$$

$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$ (you find the reciprocal of a fraction by swapping the top and bottom over)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$$

Fractional powers:
Fractional powers correspond to roots:

$a^{1/2} = \sqrt{a}$ $a^{1/3} = \sqrt[3]{a}$ $a^{1/4} = \sqrt[4]{a}$

In general:

$$a^{1/n} = \sqrt[n]{a}$$

Therefore:

$8^{1/3} = \sqrt[3]{8} = 2$ $25^{1/2} = \sqrt{25} = 5$ $10000^{1/4} = \sqrt[4]{10000} = 10$

A more general fractional power can be dealt with in the following way: $a^{m/n} = \left(a^{1/n}\right)^m$

So $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

$$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \left(\sqrt{\frac{36}{25}}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$$

Exercise 7B:

Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(\frac{1}{9}\right)^{1/2}$

4) 5^{-2}

5) 18^{0}

6) 7^{-1}

7) $27^{2/3}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-2/3}$

10) $(0.04)^{1/2}$

11) $\left(\frac{8}{27}\right)^{2/3}$

12) $\left(\frac{1}{16}\right)^{-3/2}$

Simplify each of the following:

13) $2a^{1/2} \times 3a^{5/2}$

14) $x^{3} \times x^{-2}$

15) $\left(x^{2}y^{4}\right)^{1/2}$

Chapter 8: SURDS

Surds are square roots of numbers which don't simplify into a whole (or rational) number: e.g. $\sqrt{2} \approx 1.414213...$ but it is more accurate to leave it as a surd: $\sqrt{2}$

General rules

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$$

But you cannot do:

$$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$$

These are NOT equal

$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = \sqrt{a^2} + \sqrt{ab} - \sqrt{ab} - \sqrt{b^2} = a - b$$

Simplifying Surds

Find the largest square numbers and simplify as far as possible

Worked Examples

$\sqrt{18} = \sqrt{2 \times 9} = \sqrt{2} \times \sqrt{9} = \sqrt{2} \times 3 = 3\sqrt{2}$ Careful - this is "3 times the square root of 2" NOT "the cube root of 2"

Rationalising the Denominator

This is a fancy way of saying getting rid of the surd on the bottom of a fraction. We multiply the fraction by the denominator (or the denominator with the sign swapped)

Worked Examples

1. *Rationalise* $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3}$ we multiply by $\frac{a}{a}$ which is the same as multiplying by 1, which means we don't fundamentally change the fraction.
2. *Rationalise* $\frac{3}{2\sqrt{5}} = \frac{3}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3 \times \sqrt{5}}{2\sqrt{5} \times \sqrt{5}} = \frac{3\sqrt{5}}{10}$
3. *Rationalise* $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{1 \times (\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2}) \times (\sqrt{5}-\sqrt{2})}$

$$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5^2} + \sqrt{2} \times \sqrt{5} - \sqrt{2} \times \sqrt{5} - \sqrt{2^2})} = \frac{\sqrt{5}-\sqrt{2}}{5 + \sqrt{10} - \sqrt{10} - 2} = \frac{\sqrt{5}-\sqrt{2}}{3}$$

4. *Rationalise* $\frac{\sqrt{2}}{3\sqrt{2}-1} = \frac{\sqrt{2}}{3\sqrt{2}-1} \times \frac{3\sqrt{2}+1}{3\sqrt{2}+1} = \frac{\sqrt{2} \times (3\sqrt{2}+1)}{(3\sqrt{2}-1) \times (3\sqrt{2}+1)}$

$$= \frac{3\sqrt{2^2} + \sqrt{2}}{(3^2\sqrt{2^2} + 3\sqrt{2} - 3\sqrt{2} - 1^2)} = \frac{3 \times 2 + \sqrt{2}}{9 \times 2 - 1} = \frac{6 + \sqrt{2}}{17}$$

Exercise 8A:
Simplify the surds

1) $\sqrt{12}$
2) $\sqrt{125}$
3) $\sqrt{48}$

Exercise 8B:
Expand and simplify

1) $\sqrt{2}\left(3+\sqrt{5}\right)$
2) $\sqrt{6}\left(\sqrt{2}+\sqrt{8}\right)$
3) $(2+\sqrt{5})(2+\sqrt{3})$
4) $(1-\sqrt{2})(1+\sqrt{3})$
5) $(8-\sqrt{2})(8+\sqrt{2})$
6) $(\sqrt{3}+\sqrt{5})(\sqrt{3}+\sqrt{5})$

Exercise 8C:
Rewrite the following expressions with rational denominators

1) $\dfrac{3}{\sqrt{5}}$

3) $\dfrac{9}{\sqrt{48}}$

4) $\dfrac{7}{\sqrt{7}-2}$

5) $\dfrac{-3}{\sqrt{5}+1}$

6) $\dfrac{\sqrt{3}-1}{\sqrt{5}}$

7) $\dfrac{\sqrt{5}-1}{\sqrt{5}+3}$

Chapter 9: Straight line graphs

Linear functions can be written in the form $y = mx + c$, where m and c are constants.
A linear function is represented graphically by a straight line, m is the gradients and c is the y-intercept of the graph.

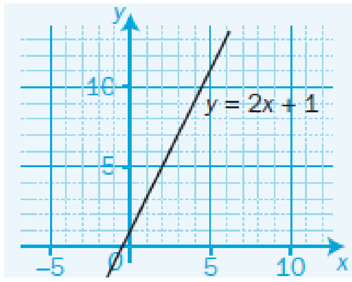
Example 1: Draw the graph of $y = 2x + 1$

Solution:

Step 1: Make a table of values

x	0	2	4
y	1	5	9

Step 2: Use your table to draw the straight line graph

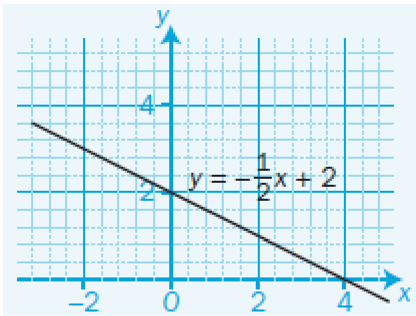


Example 2: Plot the straight line using the gradient and y intercept

Solution:

Step 1: Mark on the y axis the y-intercept = 2

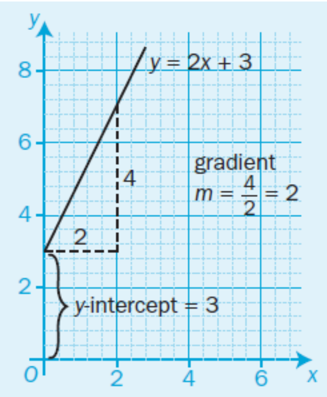
Step 2: The gradient= $-\frac{1}{2}$ so start from the y– intercept for every 1unit across to the right go down by half a unit and mark a second point there.

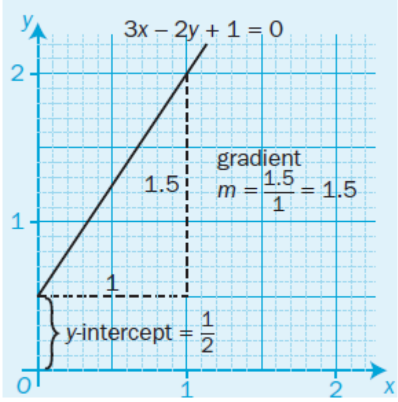


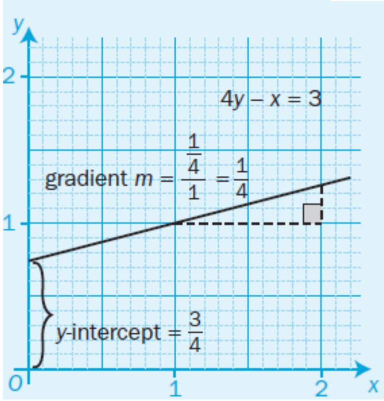
Step 3: Join the y intercept with the new point with a line and extend form both sides.

.
Here are some examples of linear functions not all of them in the form $y = mx + c$. You need to be confident into rearranging the functions making y the subject in order to identify the gradient and y-intercept.

$y = \mathbf{2}x + \mathbf{3}$	$3x - 2y + 1 = 0$	$4y - x = 3$
	so $y = \frac{3}{2}x + \frac{1}{2}$	so $y = \frac{1}{4}x + \frac{3}{4}$
gradient= 2	gradient= $\frac{3}{2}$	gradient= $\frac{1}{4}$
y-intercept= 3	y-intercept= $\frac{1}{2}$	y-intercept= $\frac{3}{4}$







To find the y-axis crossing, substitute $x = 0$ into the linear equation and solve for y.
To find the x-axis crossing, substitute $y = 0$ into the linear equation and solve for x.

Example 3: Rewrite the equation $3y - 2x = 5$ into the form $y = mx + c$, find the gradient and the y-intercept

Solution:

Step 1: Add $2x$ to both sides (so that the x term is positive): $3y = 5 + 2x$

Step 2: Divide by 3 both sides: $y = \frac{2}{3}x + \frac{5}{3}$

Step 3: Identify the gradient and y-intercept gradient= $\frac{2}{3}$ y-intercept= $\frac{5}{3}$

Example 4: Find the gradient of the line which passes through the points A (1, 4) and B (-3, 2)

Solution:

Step 1: Use the x and y values of A (x_1, y_1) and B (x_2, y_2) $m = \frac{2-4}{-3-1} = \frac{-2}{-4} = \frac{1}{2}$

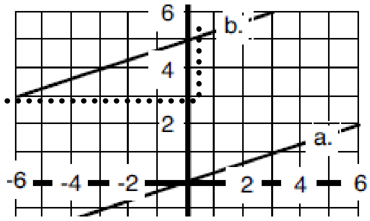
Step 2: find the gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

Finally you need to be able to find the equation of a line from a graph.

Example 5: Find the equation of the straight line which passes through the point (1, 3) and has gradient 2

Solution:

<u>Step 1</u>: Find where the line crosses the y axis.
This is the *y* intercept, *c*.



Line crosses y axis at 5,
so y-intercept c=5

<u>Step 2</u>: Draw a triangle below the line from the intercept to a point you know

And work out the gradient between the two points $m = \frac{y_2 - y_1}{x_2 - x_1}$

Gradient triangle from (-6,3) to (0,5) so $m = \frac{5-3}{0--6} = \frac{2}{6} = \frac{1}{3}$

<u>Step 3</u>: Write in the form $y = mx + c$ $y = \frac{1}{3}x + 5$

Exercise 9A: Plot the graph of each function taking the given values

a) $y = x - 3$ (x = -2 to 4)

b) $y = -x + 4$ (x = -2 to 5)

c) $y = 2x - 3$ (x = -1 to 5)

d) $y = -3x + 5$ (x = -2 to 3)

Exercise 9B:
Rewrite the equations below into the form $y = mx + c$, find the gradient and the y-intercept

a) $3x - 2y - 2 = 0$ b) $x + 2y - 8 = 0$ c) $5 = 4x - 2y$

Then plot the graph of each equation

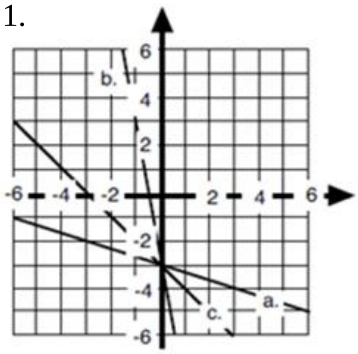
Exercise 9C:
Work out the gradient between the sets of coordinates

a) A (0, 2) and B(3, 6)
b) A (1, 0) and B(3, -2)
c) A (1, -3) and B(2, -4)
d) A (-4, 2) and B(3, 5)
e) A (1, 0.5) and B(5, -2)
f) A (-7, -3) and B(-2, -6)

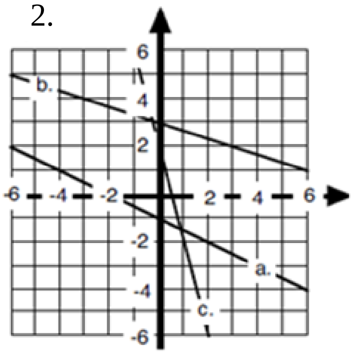
Exercise 9D:

Find the equation of these lines in the form

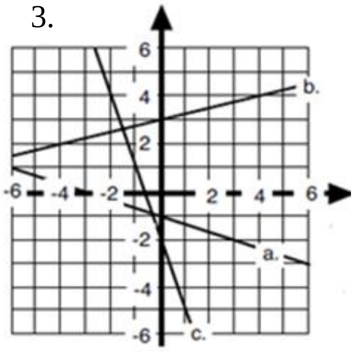
1.



2.



3.



Chapter 10: Statistics

Calculations

Means from frequency distributions

Example

The age of group of 30 people is shown in the table below

Age	Frequency	Age x f
11	7	77
12	2	24
13	3	39
14	5	70
15	10	150
16	3	48
		408

Mean age = $\frac{408}{30}$ = 13.6

Means from grouped data

Find the mid-point of each group and then multiply by frequency. Sum and then divide by total frequency

Example

The table below shows the heights of a group of students.

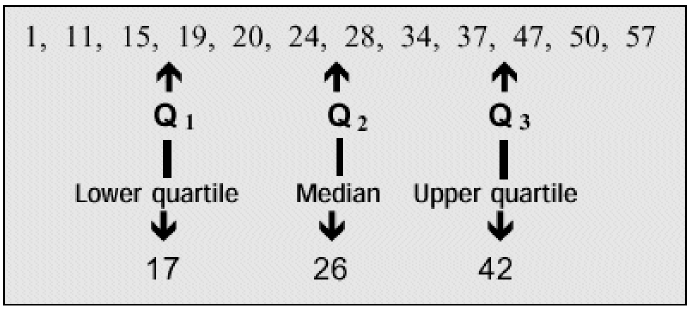
Heights	Frequency (f)	Mid-point (m)	f x m
$100 \leq h < 110$	5	105	525
$110 \leq h < 120$	7	115	805
$120 \leq h < 130$	3	125	375
	15		1705

Mean height = $\frac{1705}{15}$ = 113.67

Averages

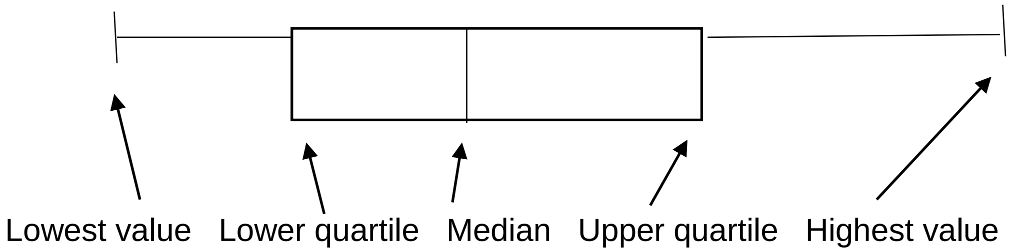
Range = difference between highest and lowest observed values

Interquartile range = difference between upper quartile (Q_3) and lower quartile (Q_1)



Charts and Graphs

Box plots



Outliers Any values 1.5 x IQR above the UQ or below the LQ are considered to be an outlier

Stem and Leaf diagrams
This is a chart to help order data.
For example
68 , 72, 56, 52, 78, 53, 64, 73
Can be represented in a stem and leaf diagram

5	2 3 6
6	4 8
7	2 3 8

Key 5 | 2 = 52

Histograms

The frequency density is the **frequency** of values **divided by** the **class width** of values.

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

The area of each block/bar represents the total of frequencies for a particular class width.

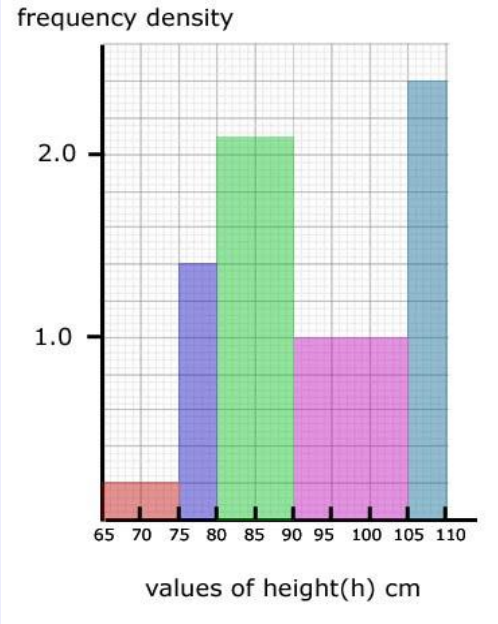
The width of the block/bar(along the x-axis) relates to the **size** of the class width. So the width of a block/bar can vary within a histogram. The frequency density is always the y-axis of a histogram.

Histograms are only used for numerical continuous data that is grouped.

<u>Example</u> Here is a table of data similar to the last one but with values of height grouped differently using inequalities.

Note: because the class is grouped using inequalities, one 'equal to and greater' and the other 'less than', the class width is a straight subtraction of the two numbers making up the class group.

class (height - h) cm	**class width**	**frequency** (f)	**frequency density**
$65 \leq h < 75$	10	2	2/10 = 0.2
$75 \leq h < 80$	5	7	7/5 = 1.4
$80 \leq h < 90$	10	21	21/10 = 2.1
$90 \leq h < 105$	15	15	15/15 = 1.0
$105 \leq h < 110$	5	12	12/5 = 2.4



Significance of area: The area on a histogram is important in being able to find the total number of values/individual results in the data.

In our histogram(from the table), the 65 to 75 block represents 2 children, the 75 to 80 block represents 7 children, and so on. So **one block square** represents **one** child.

If we count the square blocks in the whole sample we get 57 - the sum of all the frequencies i.e. the total number of children taking part - the number of individual results.

Probability

Mutually Exclusive events – Events that cannot happen at the same time
Independent events – The probability of one event is not affected by the probability of another event.
Exhaustive events – A set of events is exhaustive if the set contains all possible outcomes.

Rules of probability

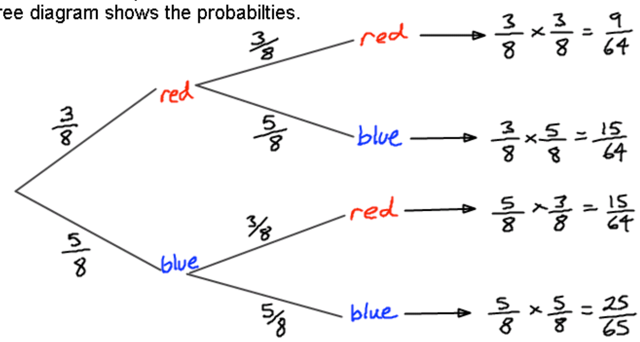
P(a or b) = P (a) + P(b)

P(a and b) = P(a) x P(b)

Tree diagrams
When completing a tree diagram remember each pair of branches must add to make 1.
As you travel along the branches to find possible outcomes you multiply the probabilities.
If the is more than one possible outcome, sum them.

Example

A bag contains 5 blue balls and 3 red balls.
A ball is picked from the bag, the colour is noted and then replaced.
A second ball is then picked.
The tree diagram shows the probabilties.



Find the probability of getting at least one red

red, red + red, blue + blue, red

$$\frac{9}{64} + \frac{15}{64} + \frac{15}{64} = \frac{39}{64}$$

Exercise 10A

1. A botany student counted the number of daisies in each of 42 randomly chosen areas of 1 m by 1 m in a large field. The results are summarised in the following stem and leaf diagram.

Number of daisies   1 | 1 means 11

1	1	2	2	3	4	4	4		(7)
1	5	5	6	7	8	9	9		(7)
2	0	0	1	3	3	3	3	4	(8)
2	5	5	6	7	9	9	9		(7)
3	0	0	1	2	4	4			(6)
3	6	6	7	8	8				(5)
4	1	3							(2)

(a) Write down the modal value of these data. **(1)**

(b) Find the median and the quartiles of these data. **(4)**

(c) Showing your scale clearly, draw a box plot to represent these data. **(4)**

2. The probability of collecting a key at the first stage is $\frac{2}{3}$, at the second stage is $\frac{1}{2}$, and at the third stage is $\frac{1}{4}$.

(a) Draw a tree diagram to represent the 3 stages of the game. **(4)**

(b) Find the probability of collecting all 3 keys. **(2)**

(c) Find the probability of collecting exactly one key in a game. **(5)**

(d) Calculate the probability that keys are not collected on at least 2 successive stages in a game. **(5)**

3. A keep-fit enthusiast swims, runs or cycles each day with probabilities 0.2, 0.3 and 0.5 respectively. If he swims he then spends time in the sauna with probability 0.35. The probabilities that he spends time in the sauna after running or cycling are 0.2 and 0.45 respectively.

(a) Represent this information on a tree diagram.

(3)

(b) Find the probability that on any particular day he uses the sauna.

(3)

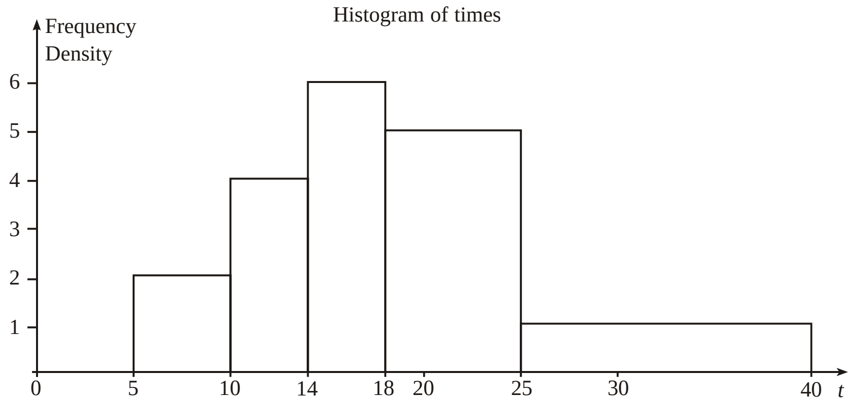
4. The total amount of time a secretary spent on the telephone in a working day was recorded to the nearest minute. The data collected over 40 days are summarised in the table below.

Time (mins)	90–139	140–149	150–159	160–169	170–179	180–229
No. of days	8	10	10	4	4	4

Draw a histogram to illustrate these data.

(Total 4 marks)

5.



The diagram above shows a histogram for the variable t which represents the time taken, in minutes, by a group of people to swim 500m.

(a) Complete the frequency table for t.

t	5–10	10–14	14–18	18–25	25–40
Frequency	10	16	24		

(2)

(b) Estimate the number of people who took longer than 20 minutes to swim 500m.

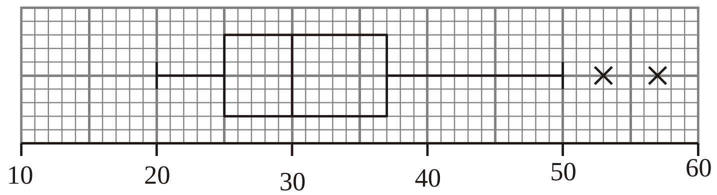
(2)

(c) Find an estimate of the mean time taken.

(4)

6. Children from school A and B took part in a fun run for charity. The times to the nearest minute, taken by the children from school A are summarised in the figure below.

School A



Time (minutes)

(a) Write down the time by which 75% of the children in school A had completed the run.

(2)

For school B the least time taken by any of the children was 25 minutes and the longest time was 55 minutes. The three quartiles were 30, 37 and 50 respectively.

(b) Draw a box plot to represent the data from school B.

(4)

(c) Compare and contrast these two box plots. (4)

SOLUTIONS TO THE EXERCISES

CHAPTER 1:

Ex A

1) $x^2 + 5x + 6$ 2) $t^2 - 3t - 10$ 3) $6x^2 + xy - 12y^2$ 4) $4x^2 + 4x - 24$ 5) $4y^2 - 1$

6) $12 + 17x - 5x^2$ 7) $49x^2 - 28x + 4$ 8) $25y^2 - 9$

CHAPTER 2

Ex A

1) 7 2) 24/7 3) 3 4) 9/5

Ex B

1) 34, 36, 38 2) 9.875, 29.625 3) 24, 48

CHAPTER 3

1) $x = 1$, $y = 3$ 2) $x = 3$, $y = 1$ 3) $a = 7$, $b = -2$

CHAPTER 4

Ex A

1) $3q(p - 3q)$ 2) $4a^3b^2(2a^2 - 3b^2)$ 3) $(y - 1)(5y + 3)$

Ex B

1) $(x + 8)(x - 2)$ 2) $(3x - 1)(x + 2)$ 3) $(2y + 3)(y + 7)$ 4) $5(2x - 3)(x + 2)$

5) $(2x + 5)(2x - 5)$ 6) $(x - 3)(x - y)$

CHAPTER 5

Ex A

1) $x = \dfrac{y+1}{7}$ 2) $x = 4y - 5$ 3) $x = 3(4y + 2)$ 4) $x = \dfrac{9y + 20}{12}$ 5) $t = \dfrac{P^2 g}{2}$

Ex B

1) $x = \dfrac{c-3}{a-b}$ 2) $x = \dfrac{3a + 2k}{k - 3}$ 3) $x = \dfrac{2y+3}{5y-2}$ 4) $x = \dfrac{ab}{b-a}$

CHAPTER 6

1) a) -1, -2 b) -5, 3

2) a) 0, -3 b) 2, -2

3) -1/2, 4/3

4) a) -5.30, -1.70 b) -1.20, 1.45 c) no solutions

CHAPTER 7

Ex A

1) b^3c^4 2) $-12n^8$ 3) $4n^5$ 4) d^2 5) a^6 6) $-d^{12}$

Ex B

1) 2 2) 3 3) 1/3 4) 1/25 5) 1 6) 1/7 7) 9 8) 9/4 9) ¼ 10) 0.2 11) 4/9 12) 64

13) $6a^3$ 14) x 15) xy^2

CHAPTER 8

ExA

1) $2\sqrt{2}$

2) $5\sqrt{3}$

3) $4\sqrt{3}$

ExB

1) $3\sqrt{2}+\sqrt{10}$

2) $\sqrt{12}+\sqrt{48}=2\sqrt{3}\$+4\sqrt{3}$

3) $4+2\sqrt{3}+2\sqrt{5}+\sqrt{15}$

4) $1+\sqrt{3}-\sqrt{2}-\sqrt{6}$

5) $64+8\sqrt{2}-8\sqrt{2}-2=62$

6) $3+\sqrt{15}+\sqrt{15}+5=8+2\sqrt{15}$

Ex C

1) $\frac{3}{\sqrt{5}}=\frac{3\sqrt{5}}{5}$

2) $\frac{9}{\sqrt{48}}=\frac{9}{4\sqrt{3}}=\frac{9\sqrt{3}}{12}=\frac{3\sqrt{3}}{4}$

3) $\frac{7}{\sqrt{7}-2}=\frac{7(\sqrt{7}+2)}{3}$

4) $\frac{-3}{\sqrt{5}+1}=\frac{-3(\sqrt{5}-1)}{4}$

5) $\frac{\sqrt{3}-1}{\sqrt{5}}=\frac{(\sqrt{3}-1)\sqrt{5}}{5}$

6) $\frac{\sqrt{5}-1}{\sqrt{5}+3}=\frac{(\sqrt{5}-1)(\sqrt{5}-3)}{-4}=\frac{5-4\sqrt{5}+3}{-4}=\frac{8-4\sqrt{5}}{-4}=-2+\sqrt{5}$

CHAPTER 9

ExB : $a)\ y=\frac{3}{2}x-1 \quad b)\ y=-\frac{1}{2}x+4 \quad c)\ y=2x-\frac{5}{2}$

ExC : $a)\,gradient=\frac{4}{3}$

$b)\,gradient=\frac{-2}{2}=-1$

$c)\,gradient=\frac{-1}{1}=-1$

$d)\,gradient=\frac{3}{7}$

$e)\,gradient=\frac{-2.5}{4}=\frac{-5}{8}$

$f)\,gradient=\frac{-3}{5}$

Ex D : **1**) a. $y = -\frac{1}{3}x - 3$ b. $y = -6x - 3$ c. $y = -x - 3$

2) a. $y = -0.5x - 1$ b. $y = -\frac{1}{3}x + 3$ c. $y = -4x + 2$

3) a. $y = -\frac{1}{3}x - 1$ b. $y = 0.25x + 3$ c. $y = -3x - 2$

CHAPTER 10

1. (a) Mode = 23 B1 1

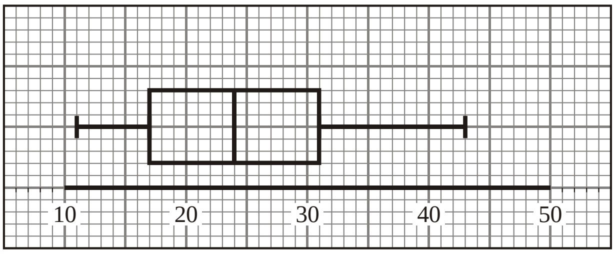
(b) For Q_1 : $\frac{n}{4} = 10.5 \Rightarrow$ 11th observation $\therefore Q_1 = 17$ B1

For Q_2 : $\frac{n}{2} = 21 \Rightarrow = \frac{1}{2}$ (21st & 22nd) observations

$\therefore Q_2 = \frac{23+24}{2} = 23.5$ M1 A1

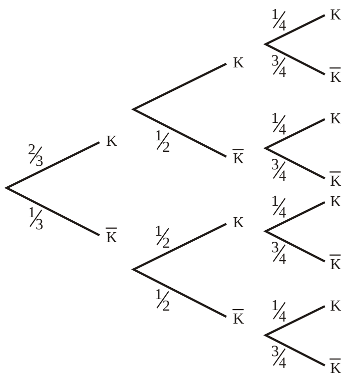
For Q_3 : $\frac{3n}{4} = 31.5 \Rightarrow$ 32nd observation $\therefore Q_3 = 31$ B1 4

(c)



Box plot M1
Scale & label M1
Q_1, Q_2, Q_3 A1
11, 43 A1 4

2. (a)
Tree with correct number of branches M1
$\frac{2}{3}, \frac{1}{3}$ A1
$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ A1
$\frac{1}{4}, \frac{3}{4} \ldots \frac{3}{4}$ A1 (4)



(b) P (All 3 Keys) = $\frac{2}{3} \times \frac{1}{2} \times \frac{1}{4} = \frac{2}{24} = \frac{1}{12}$ M1 A1 2

$\frac{1}{12}$; $0.08\dot{3}$; 0.0833

(c) P(exactly 1 key) = $\left(\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}\right)$ 3 triples added M1

$= \underline{\frac{10}{24} = \frac{5}{12}}$

Each correct

$\frac{10}{24}$; $\frac{5}{12}$; $0.41\dot{6}$; 0.417 A1 A1 A1 A1 5

(d) P (Keys not collected on at least 2 successive stages)

$$=\left(\frac{2}{3}\times\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{3}{4}\right)$$

3 triples added M1
Each correct A1 A1 A1

$$=\frac{10}{24}=\frac{5}{12}$$ A1 5

$\frac{10}{24}$; $\frac{5}{12}$; $0.41\dot{6}$; 0.417

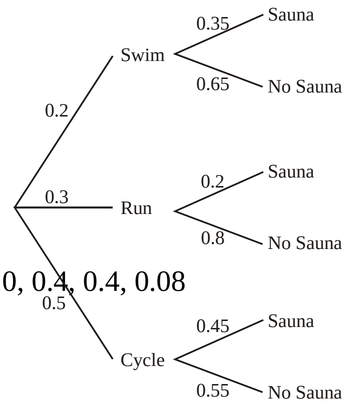
Alternative:
1 – P (Keys collected on at least 2 successive stages) M1

$$= 1 - \left\{\left(\frac{2}{3}\times\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{2}{3}\times\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{1}{4}\right)\right\}$$ A1 A1 A1

$$= \frac{5}{8}$$ A1 5

3. (a) Tree with correct number of branches M1
0.2, 0.3, 0.5 A1
All correct A1 3

(b) P(used sauna) = (0.2 × 0.35) + (0.3 × 0.2) + (0.5 × 0.45) M1 A1
= 0.355 A1 3



4. Frequency densities: 0.16, 1.0, 1.0, 0.4, 0.4, 0.08 M1, A1

Histogram: Scale and labels B1

Correct histogram B1
[4]

5. (a) 18-25 group, area = 7 × 5 = 35 B1
25-40 group, area = 15 × 1 = 15 B1 2

(b) (25 – 20) × 5 + (40 – 25) × 1 = 40 M1A1 2

5 × 5 is enough evidence of method for M1.
Condone 19.5, 20.5 instead of 20 etc.
Award 2 if 40 seen.

(c) Mid points are 7.5, 12, 16, 21.5, 32.5 M1
Σf = 100 B1

$$\frac{\sum ft}{\sum f}=\frac{1891}{100}=18.91$$ M1A1 4

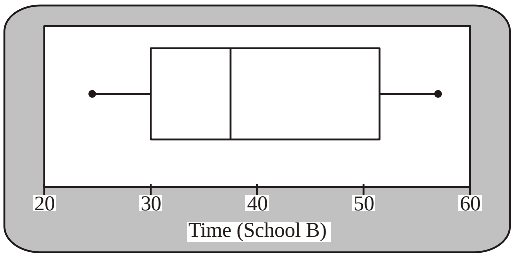
Use of some mid-points, at least 3 correct for M1.

Their $\frac{\sum ft}{\sum f}$ for M1 and anything that rounds to 18.9 for A1.

6.

(a) 37 (minutes) B1

(b)



Box & median & whiskers	M1	
Sensible scale	B1	
30, 37, 50	B1	
25, 55	B1	4

(c)

Children from school A generally took less time	B1	
50% of B ≤ 37 mins, 75% of A < 37 mins (similarly for 30)	B1	
Median / Q1 / Q3 / of A < median / Q1 / Q3 / of (1 or more)	B1	
A has outliers, (B does not)	B1	4
Both positive skew		
IQR of A < IQR of B, range of A > range of B		

*Any **correct** 4 lines*